Categorical Decisions

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The Ratio Rule

$$P(i) = \frac{v_i}{\sum_{j=1}^{n} v_j}$$

- Constant-Ratio Rule (Clarke, 1957)
- Luce Choice Axiom (Luce, 1959)

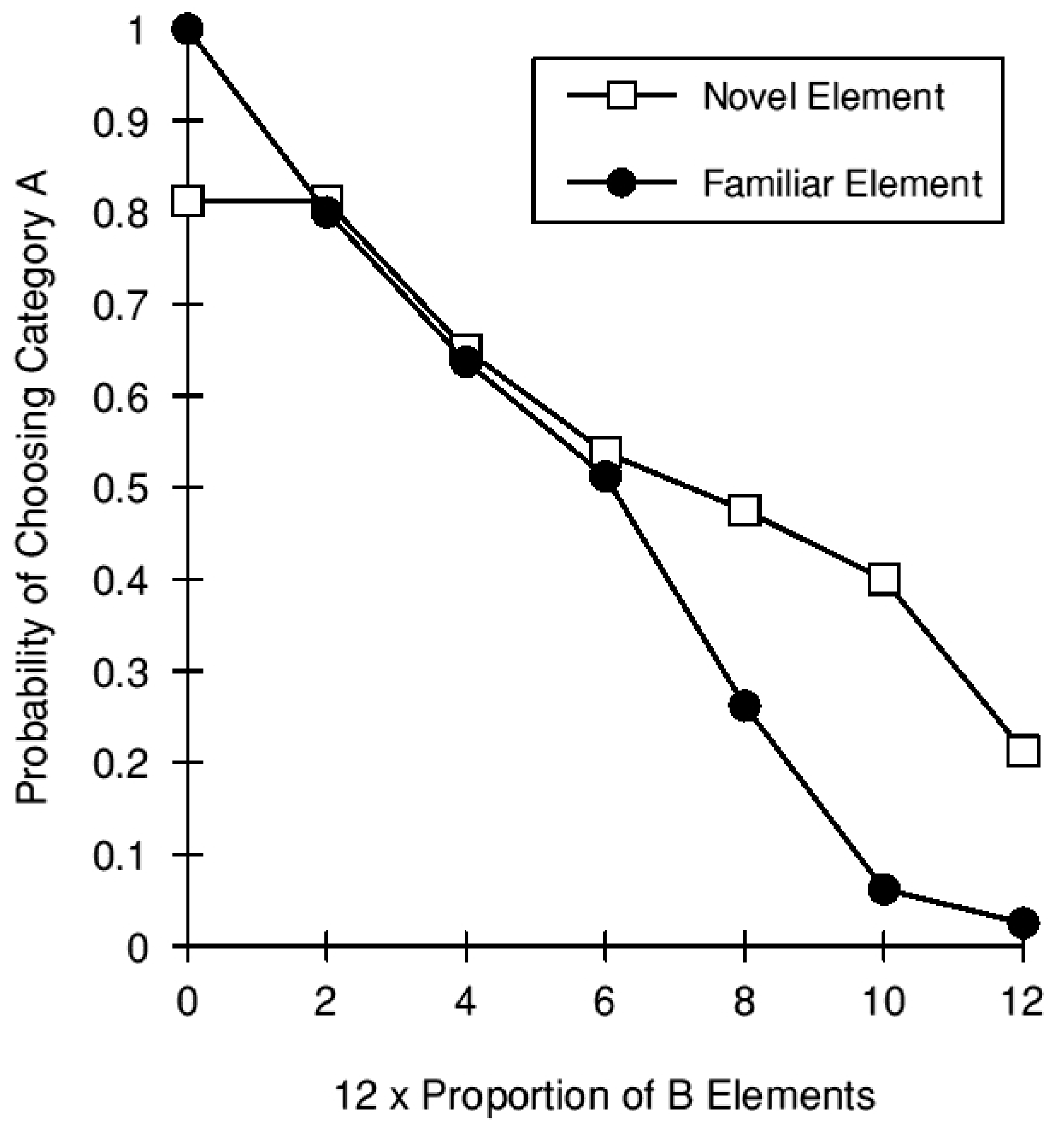
$$P(A:A,B) = \frac{v_A}{v_A + v_B}$$

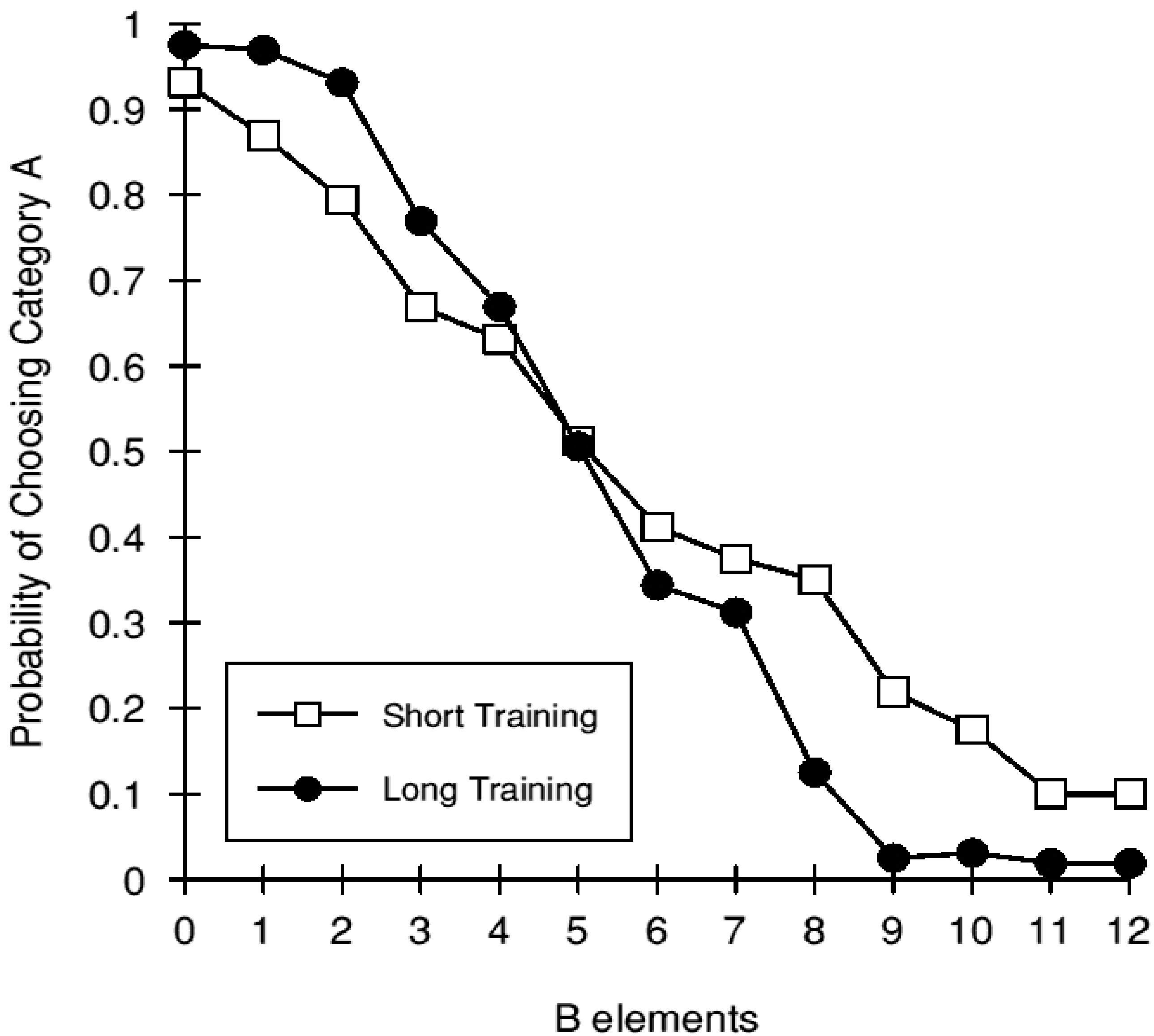
$$P(A:A,B) = \frac{12}{4+12} = 0.75$$

$$P(A:A,B) = \frac{9}{3+9} = 0.75$$

Generalisation Test

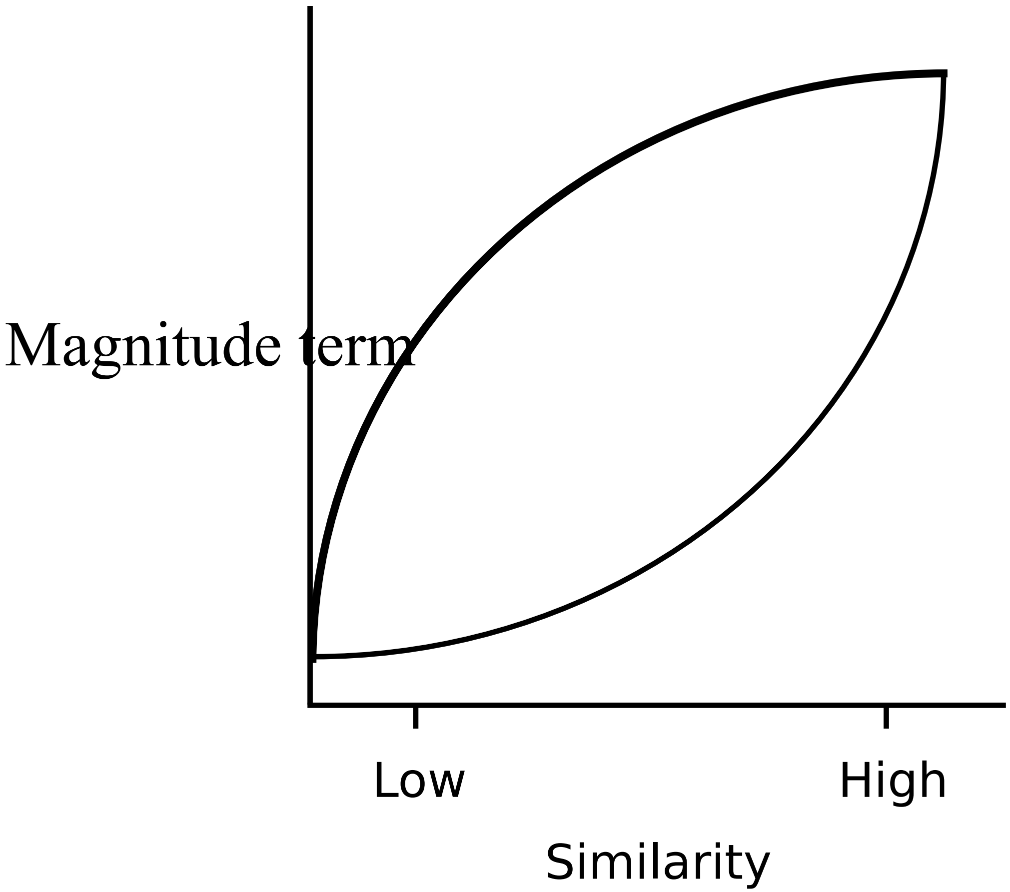
	Familiar-element Test Exemplars			Novel-element Test Exemplars		
Proportion	*A*	*B*	*Novel*	*A*	*B*	*Novel*
0/12	12	0	0	6	0	6
2/12	10	2	0	5	1	6
4/12	8	4	0	4	2	6
6/12	6	6	0	3	3	6
8/12	4	8	0	2	4	6
10/12	2	10	0	1	5	6
12/12	0	12	0	0	6	6


Novel Element
Familiar Element
Probability of Choosing Category A
12 x Proportion of B Elements
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12



Probability of Choosing Category A
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10 11 12
B elements
Short Training
Long Training

Non-linear Magnitude Functions



Background Noise Constant

$$P(i) = \frac{v_i + X}{\sum_{j=1}^{n} v_j + nX}$$

- Aitken (1996)
- Nosofsky & Zaki (1998)

Central Assumption

- The magnitude term for a specific category produced by a stimulus is a **univariate** function of the number of category-appropriate elements that stimulus contains.

Full-set / Sub-set Relationship

- Train on three categories
- Test on examples where magnitude for one category is fixed:

A	4	4	4	4	4	4	4	4	4
B	8	7	6	5	4	3	2	1	0
C	0	1	2	3	4	5	6	7	8

- Ask subjects one of two questions:
  - Category A, B or C?
  - Category B or C (A disallowed)?

Dependent Measure I

- Probability of choosing the fixed-magnitude alternative:

$$P(A:A,B,C)=\frac{v_A}{v_A+v_B+v_C}$$

Dependent Measure II

$$q = \frac{\Pi(B:\ B, X) - \Pi(B:\ A, B, X)}{\Pi(B : A, B, X)}$$

$$q = \frac{\dfrac{v_B}{v_B + v_C} - \dfrac{v_B}{v_A + v_B + v_C}}{\dfrac{v_B}{v_A + v_B + v_C}} \qquad q = \frac{v_A}{v_B + v_C}$$

A Prediction of the Ratio Rule

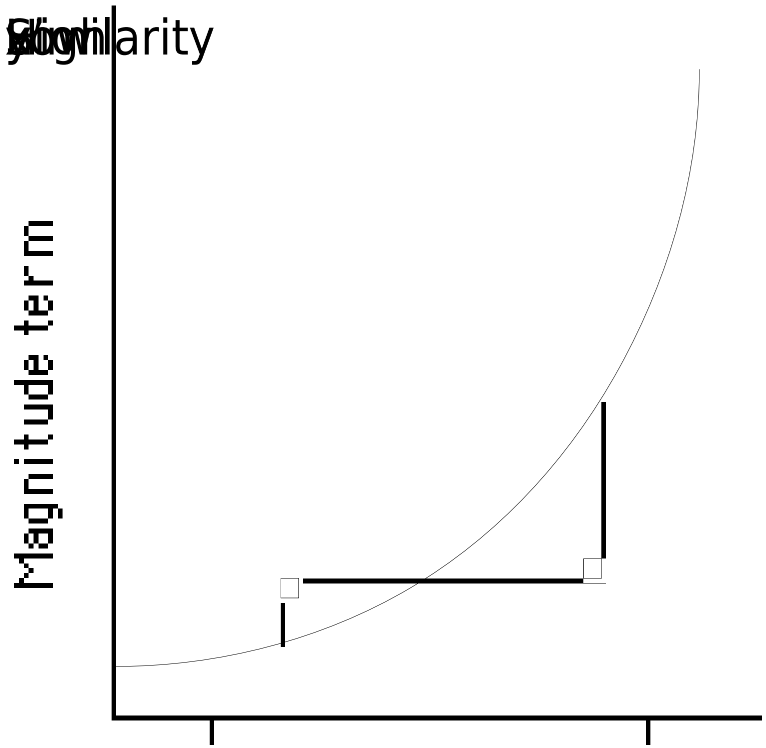
- Compare the two equations:

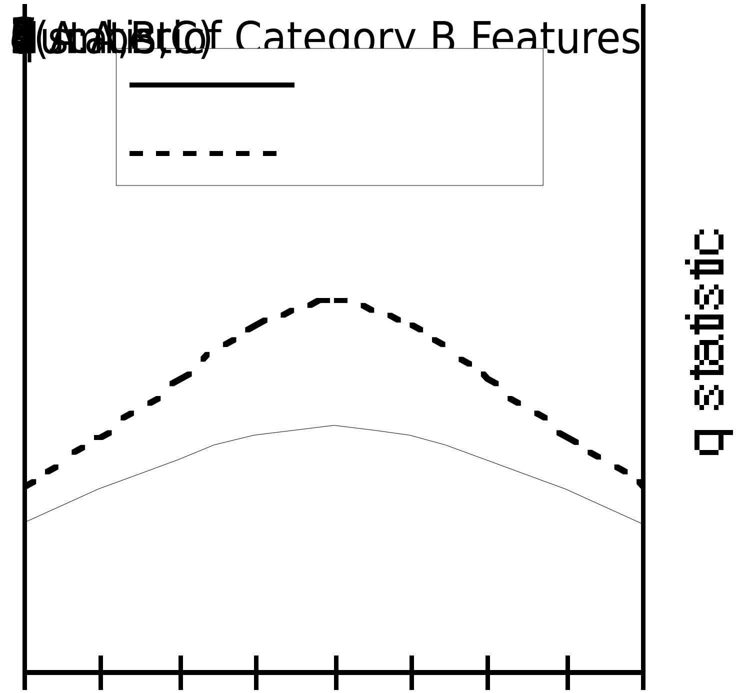
$$P(A:A,B,C) = \frac{v_A}{v_A + v_B + v_C}$$

$$q = \frac{v_A}{v_B + v_C}$$

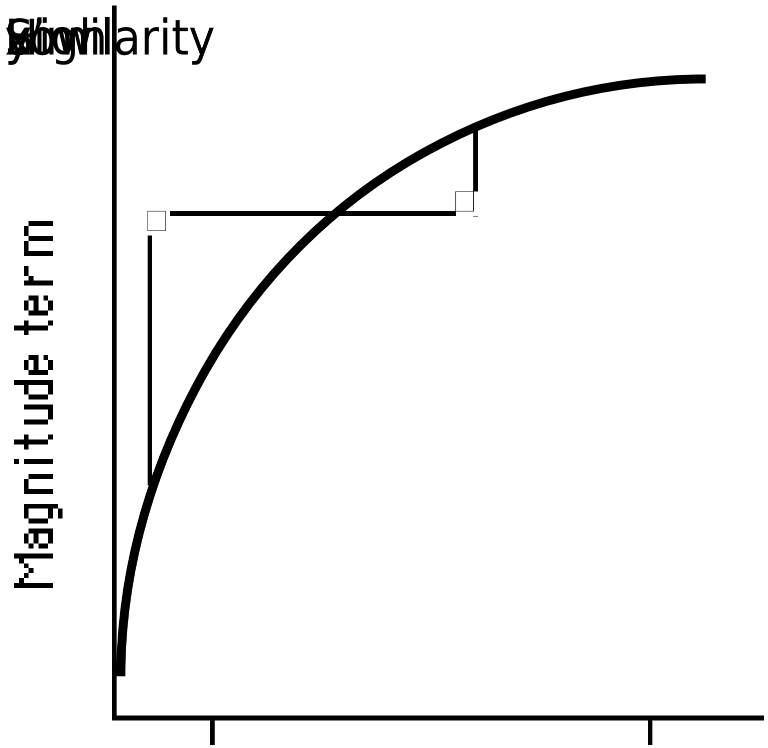
- *Any given change in* $(\mathbf{v_B+v_C})$ *must produce the same direction of change in* **q** *and* **P(A:A,B,C)**

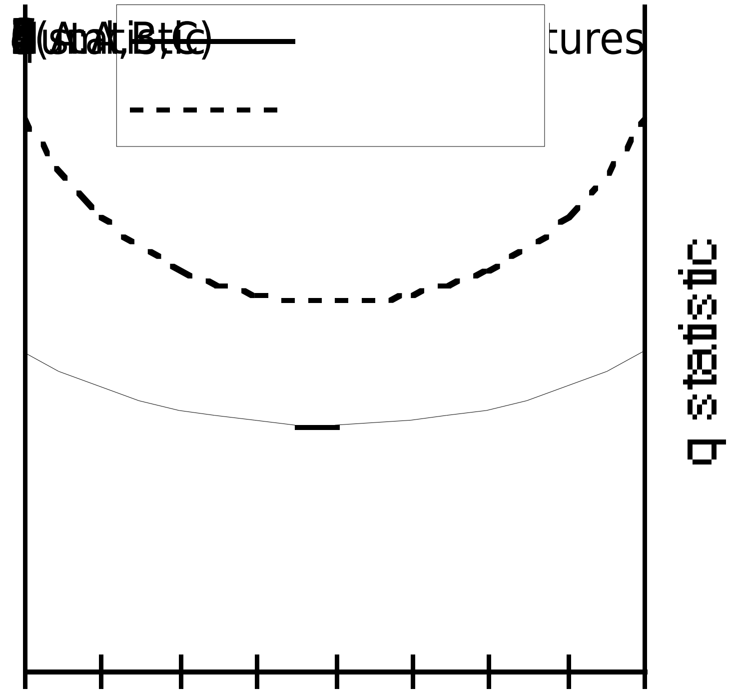
Further Predictions

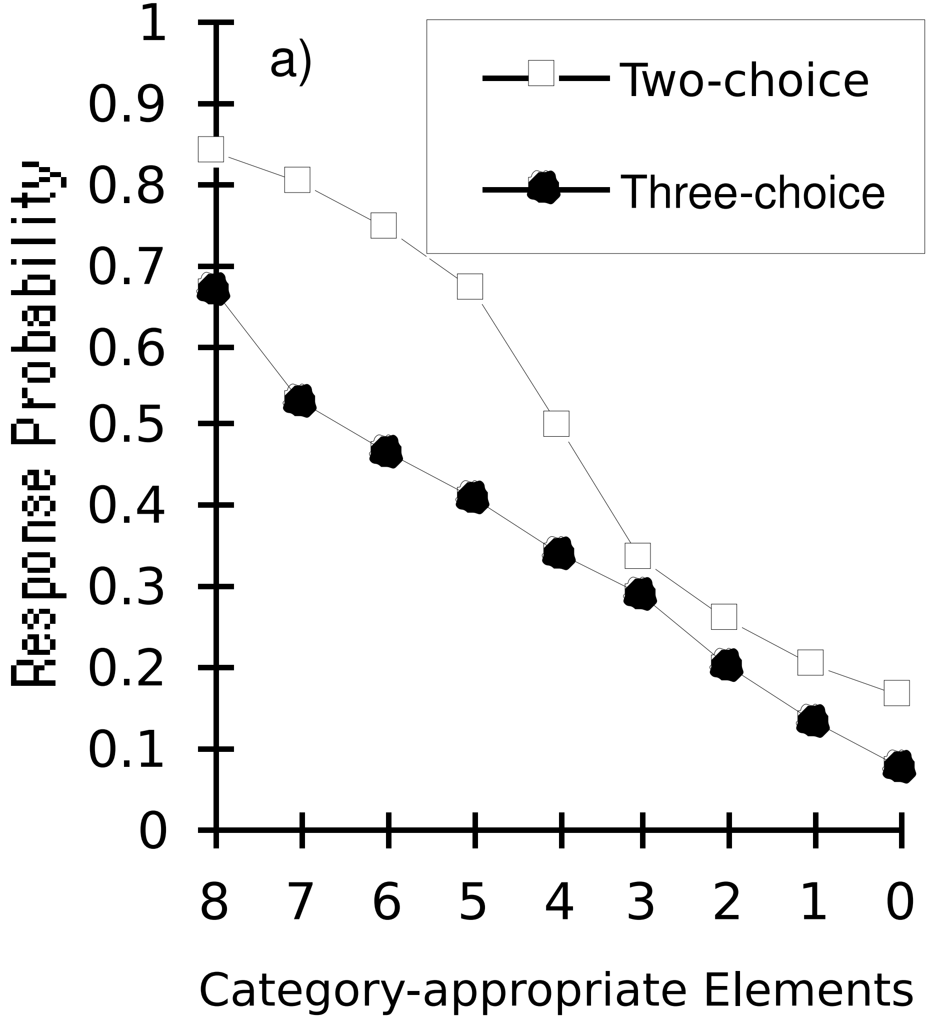




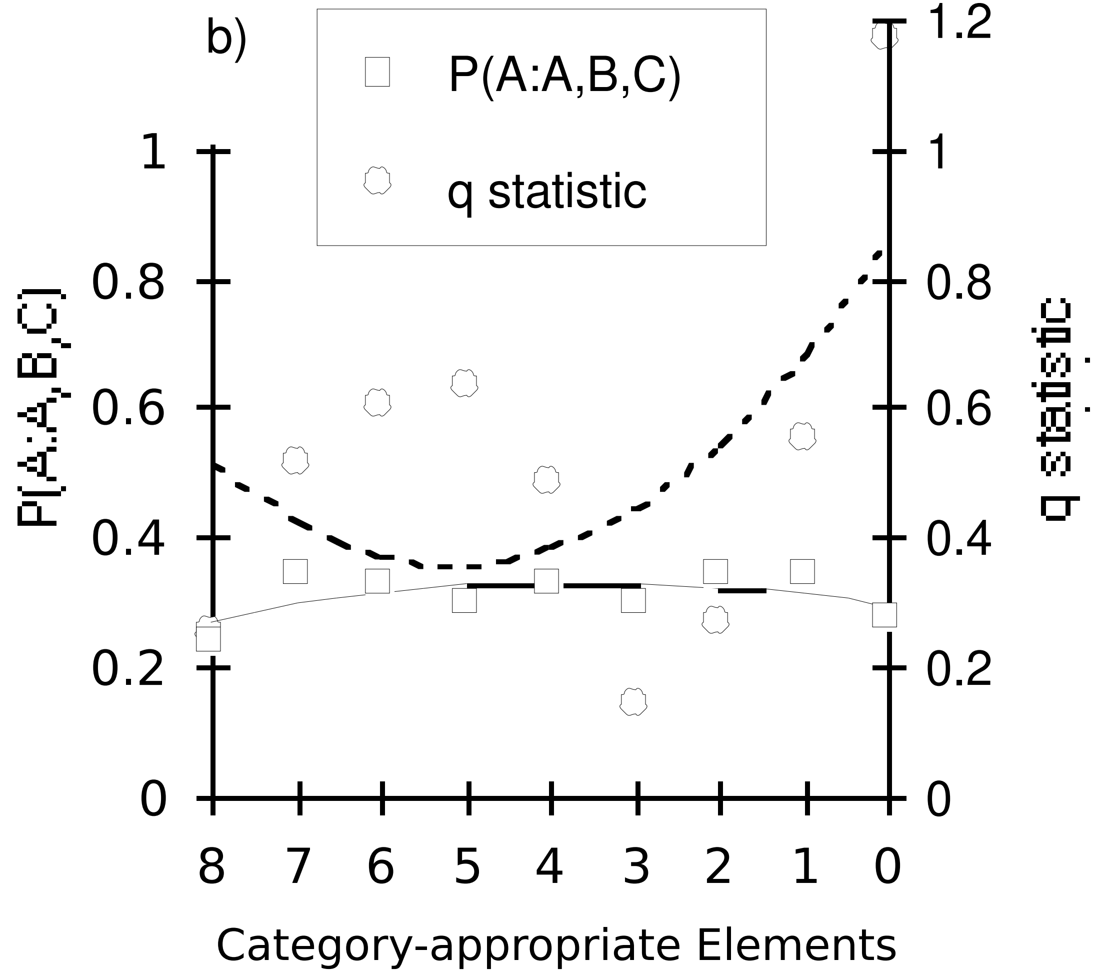
Further Predictions



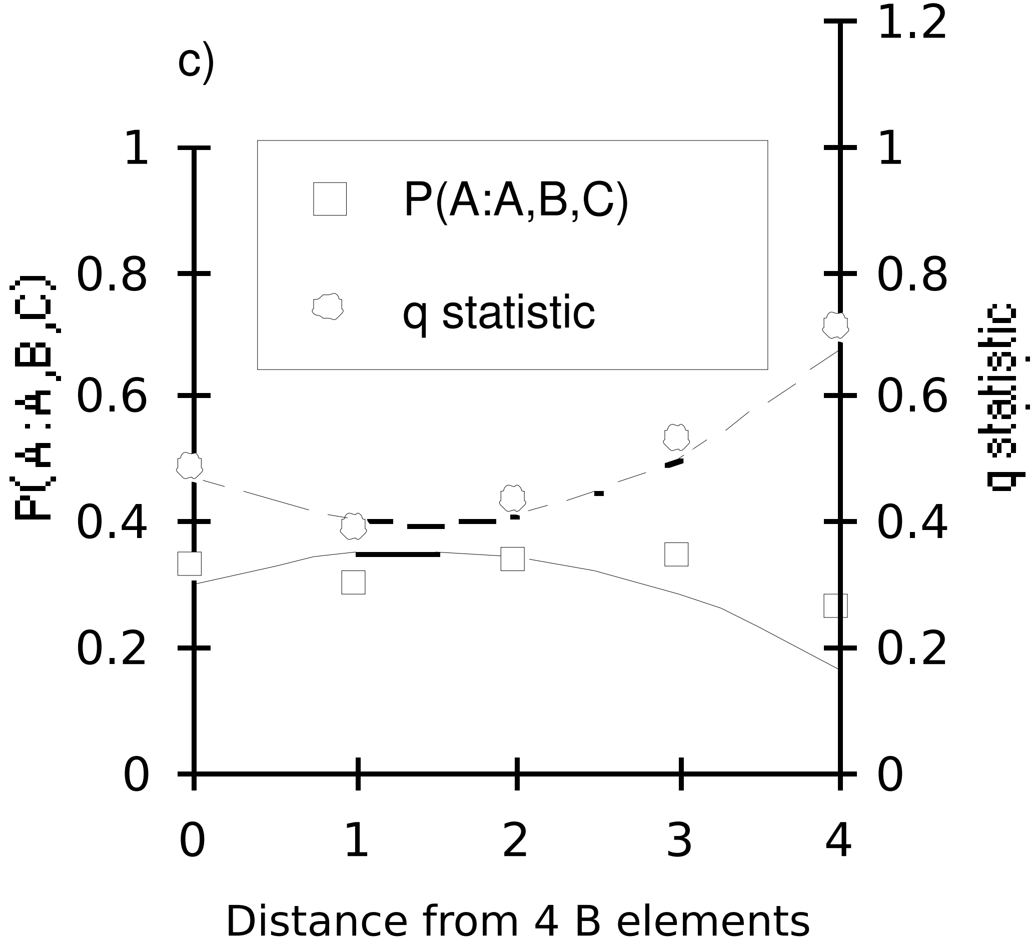




a)
Two-choice
Three-choice
Response Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8
7
6
5
4
3
2
1
0
Category-appropriate Elements



b)
P(A:A,B,C)
q statistic
P(A:A,B,C)
q statistic
0
0.2
0.4
0.6
0.8
1
1.2
8
7
6
5
4
3
2
1
0
Category-appropriate Elements



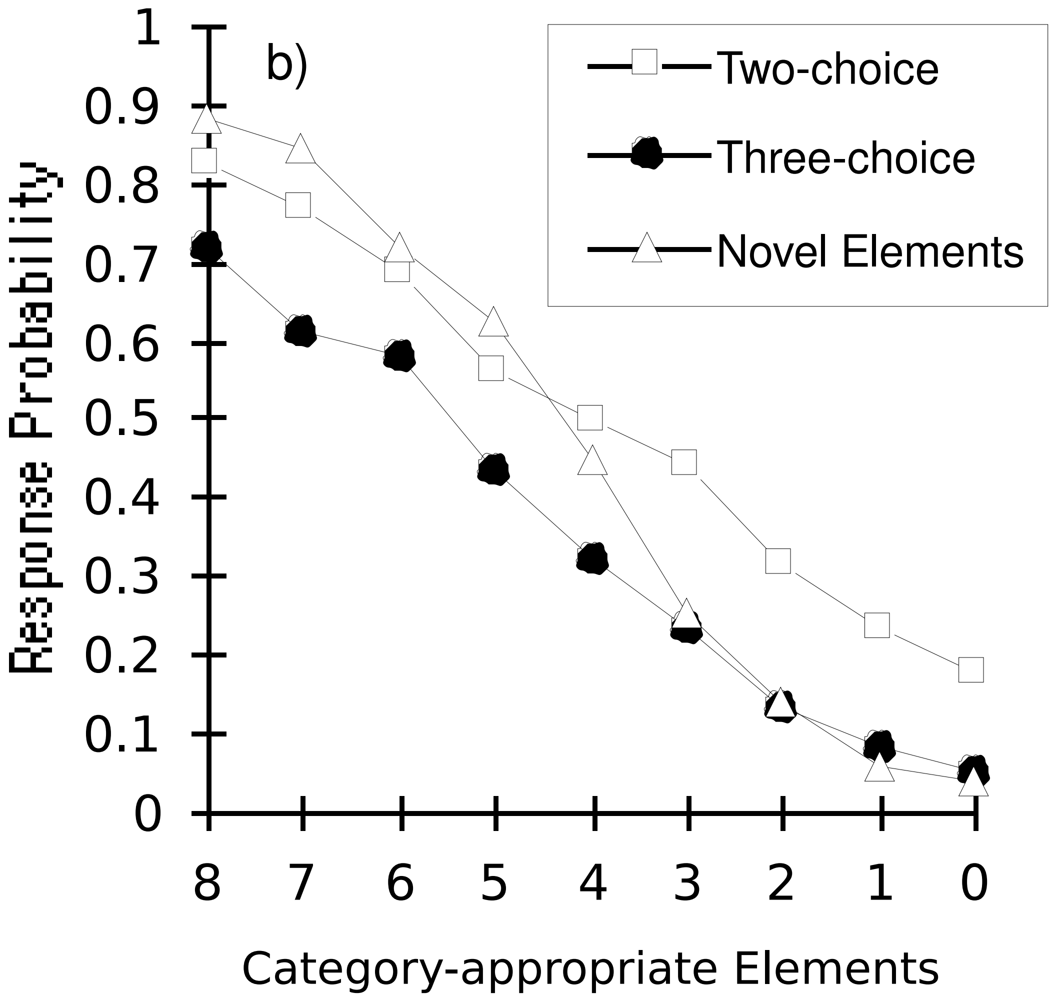
c)
P(A:A,B,C)
q statistic
1
0.8
0.6
0.4
0.2
0
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
Distance from 4 B elements

Replication and Extension

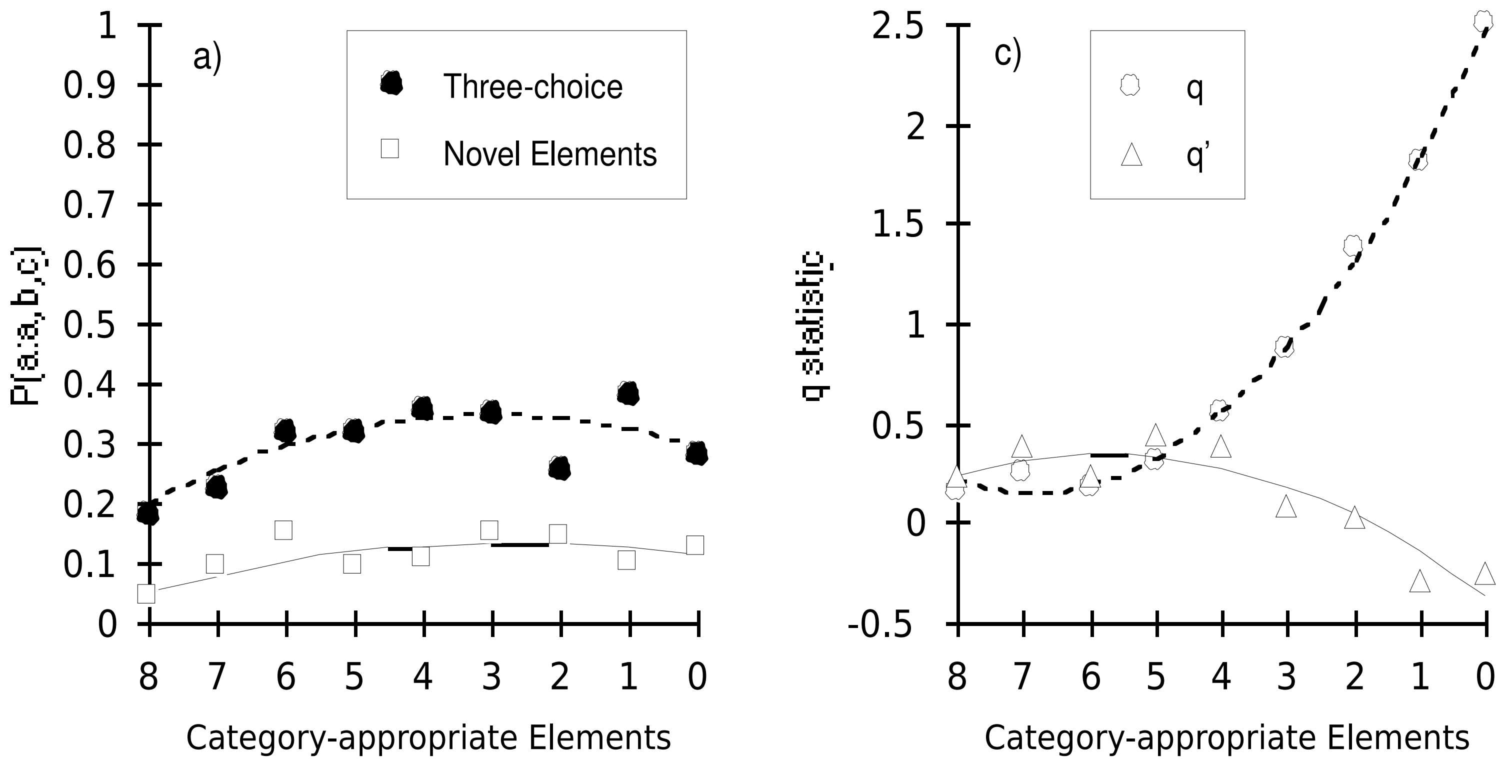
- Extra three-choice condition - category A elements replaced with novel elements at test.

$$q' = \frac{P(b:a,b,c)' - P(b:a,b,c)}{P(b:a,b,c)} = \frac{\dfrac{v_B}{v_N + v_B + v_C} - \dfrac{v_B}{v_A + v_B + v_C}}{\dfrac{v_B}{v_A + v_B + v_C}}$$

$$q' = \frac{\varpi_A - \varpi_N}{\varpi_B + \varpi_X + \varpi_N}$$



b)
Two-choice
Three-choice
Novel Elements
Response Probability
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
8
7
6
5
4
3
2
1
0
Category-appropriate Elements



a)
Three-choice
Novel Elements
P[a:a,b,c]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8
7
6
5
4
3
2
1
0
Category-appropriate Elements
c)
q
q'
q statistic
2.5
2
1.5
1
0.5
0
-0.5
8
7
6
5
4
3
2
1
0
Category-appropriate Elements

Modification of the Ratio Rule?

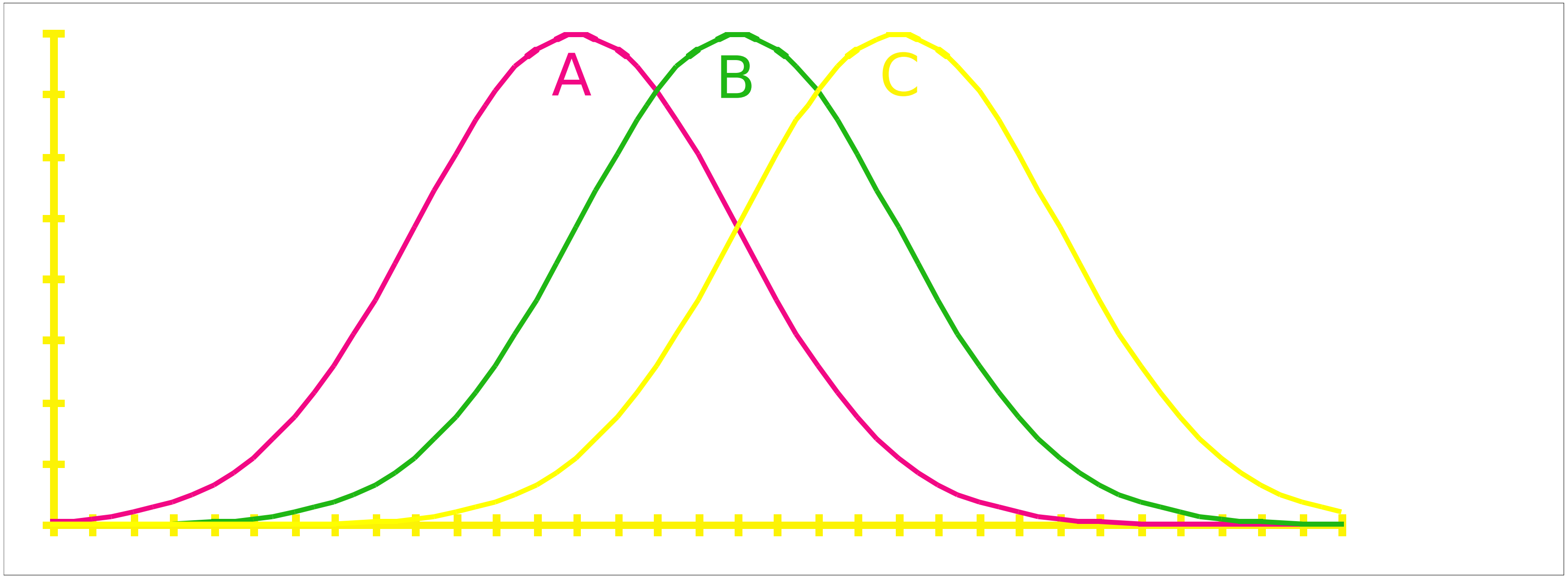
- Restle modification

$$P(A:A,B)=\frac{v_A - o_{AB}}{v_A + v_B - 2o_{AB}}$$

- Tversky extension

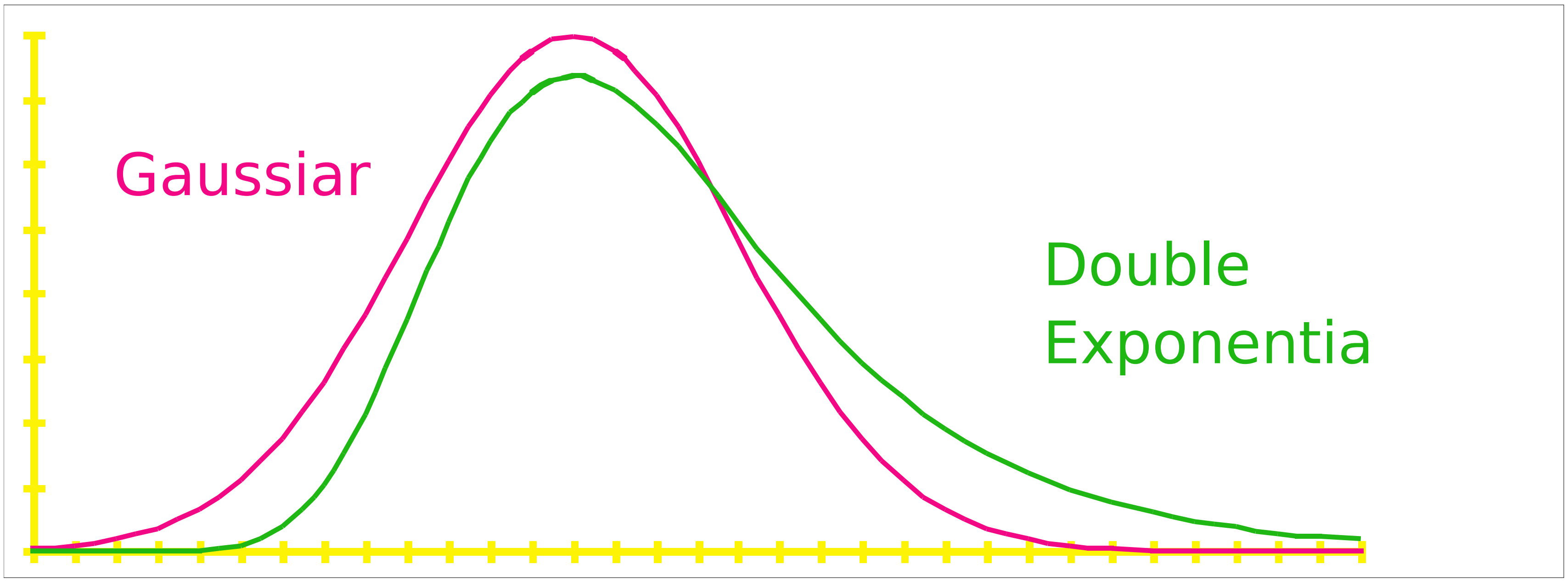
Thurstonian Choice

- Pick the biggest from noisy alternatives

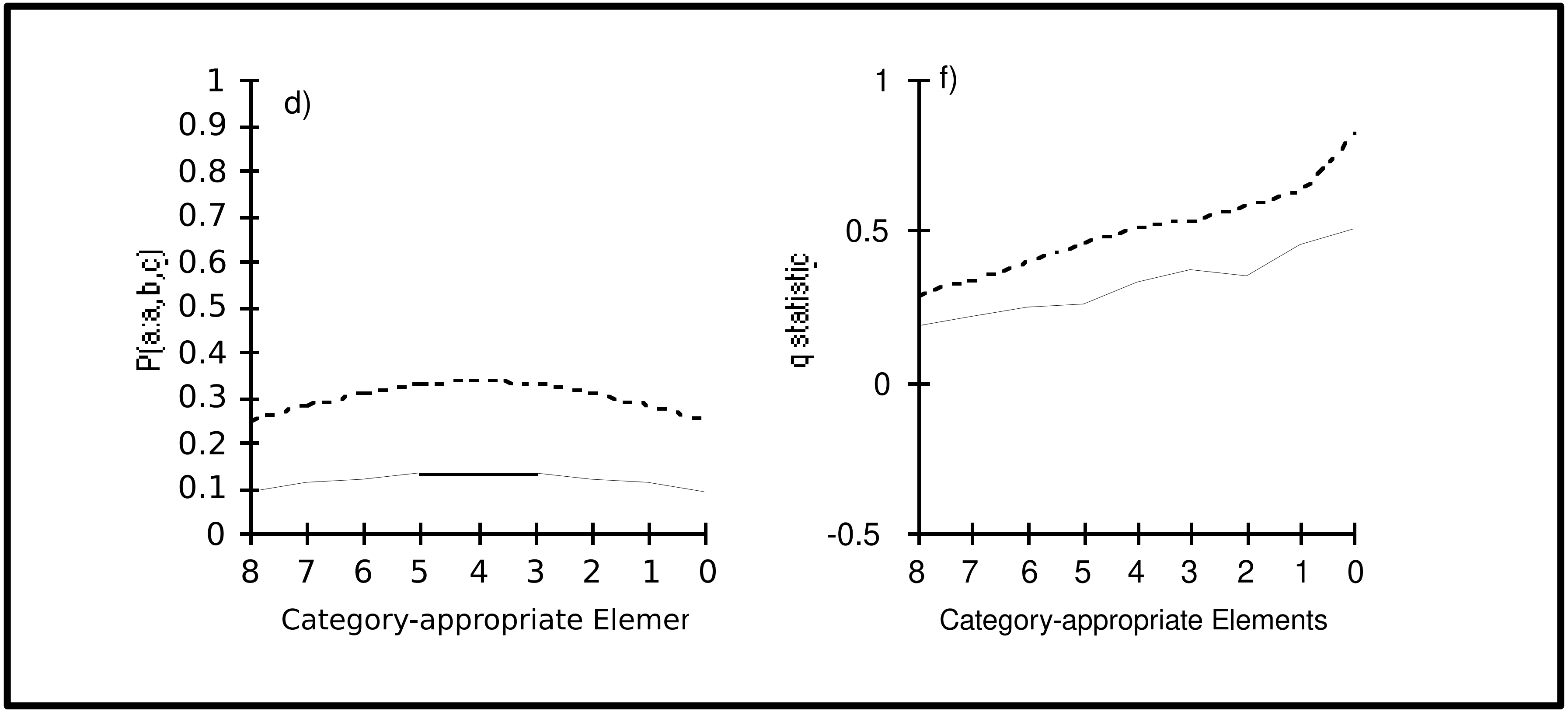


Thurstonian Choice

- Ratio Rule and Thurstonian Choice not equivalent

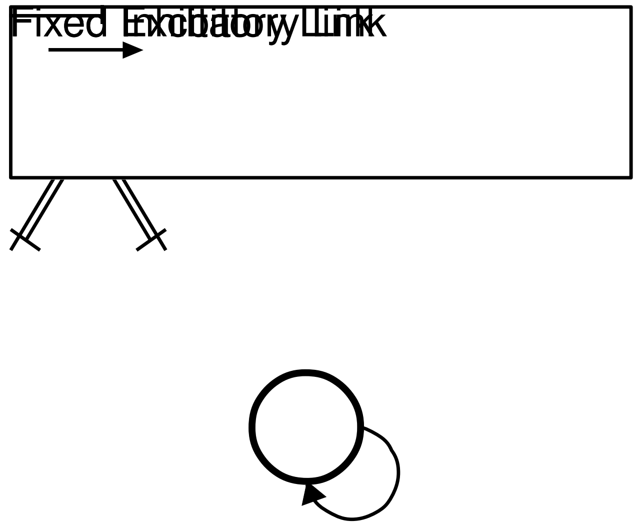


Simulation

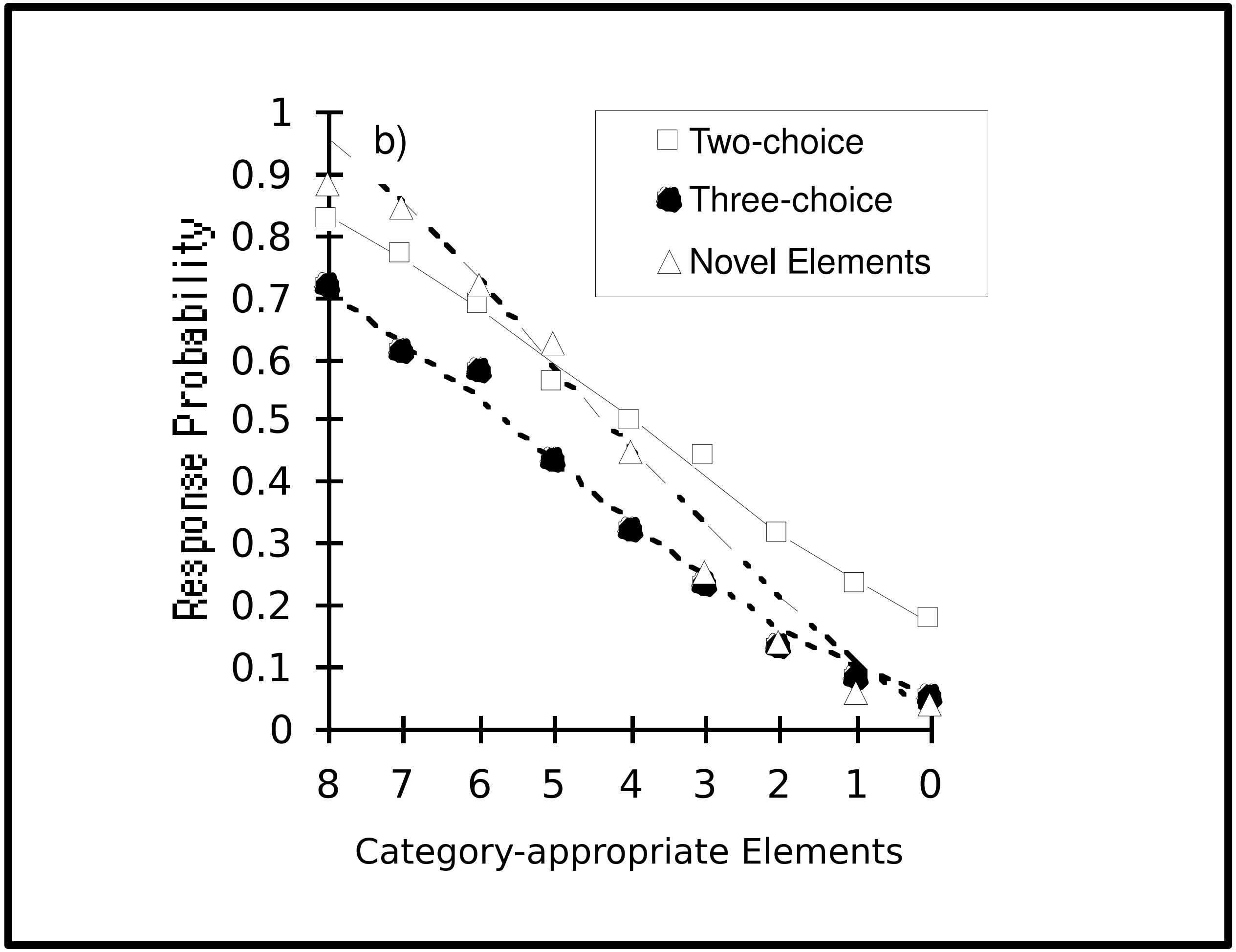


d)
P[a:a,b,c]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8 7 6 5 4 3 2 1 0
Category-appropriate Elemer
f)
q statistic
1
0.5
0
-0.5
8 7 6 5 4 3 2 1 0
Category-appropriate Elements

Connectionist Implementation

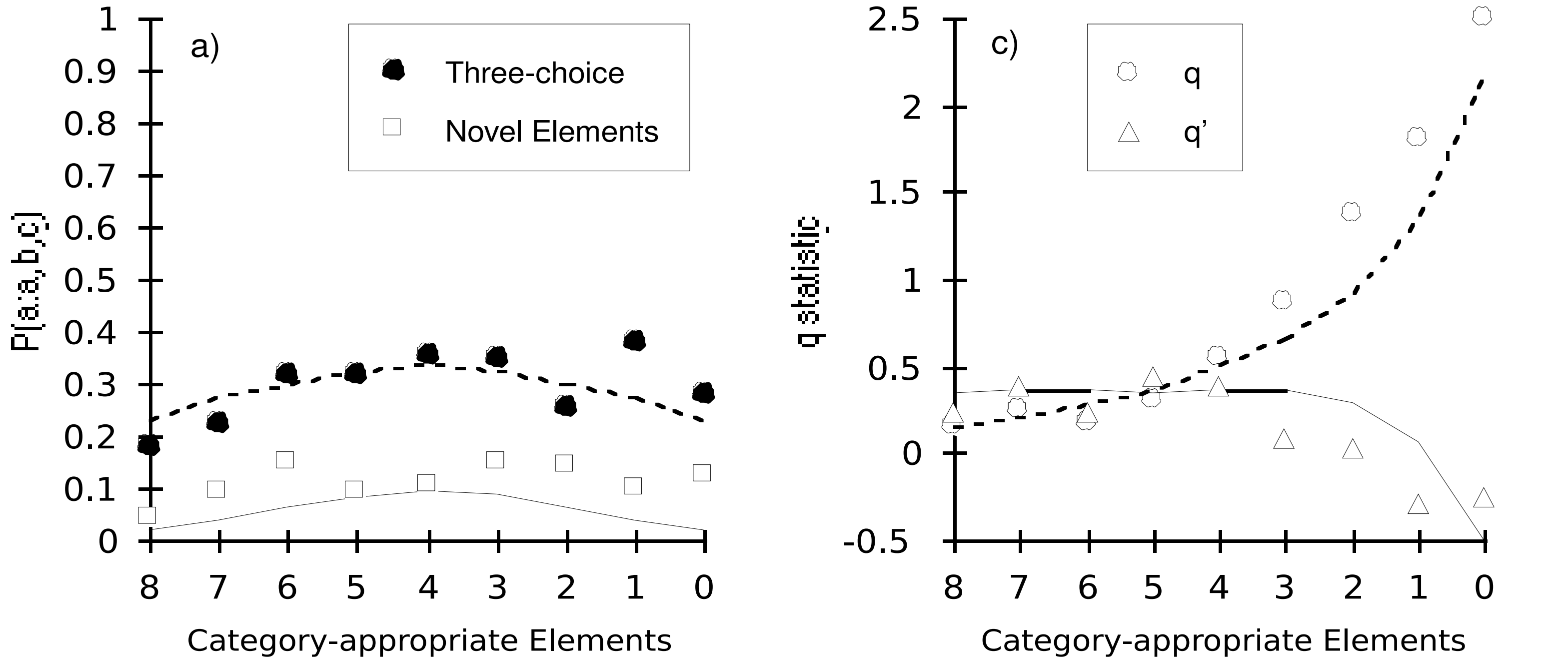


Simulation



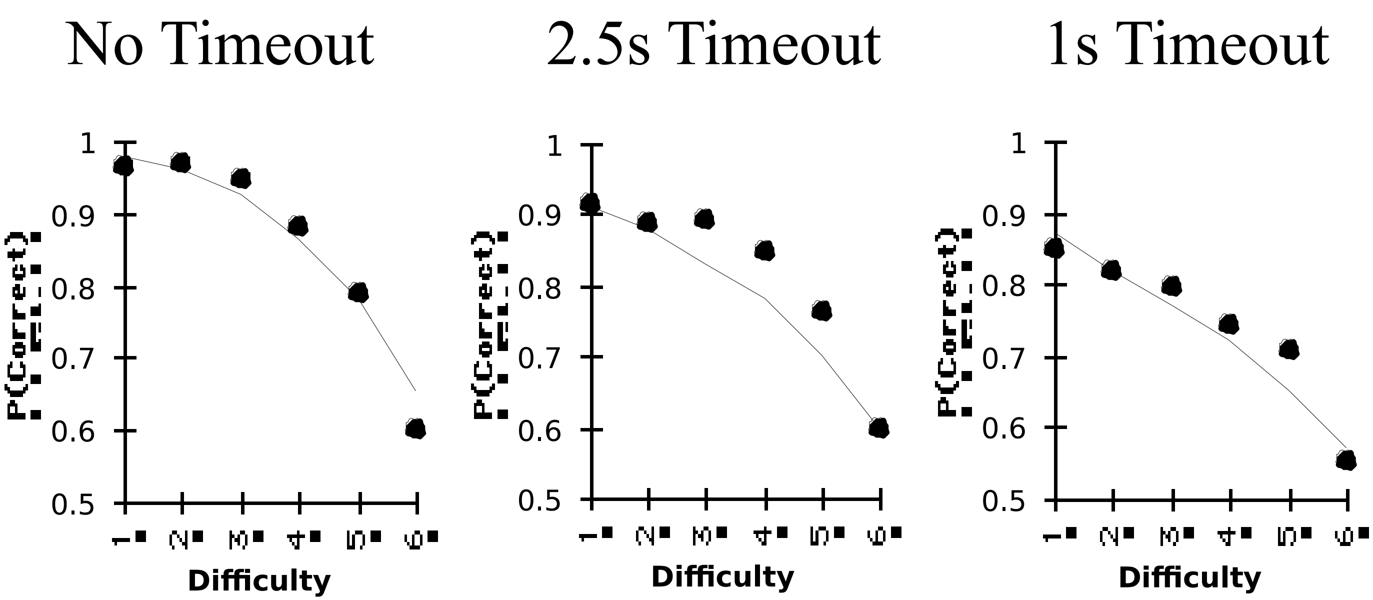
b)
Two-choice
Three-choice
Novel Elements
Response Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8
7
6
5
4
3
2
1
0
Category-appropriate Elements

Simulation

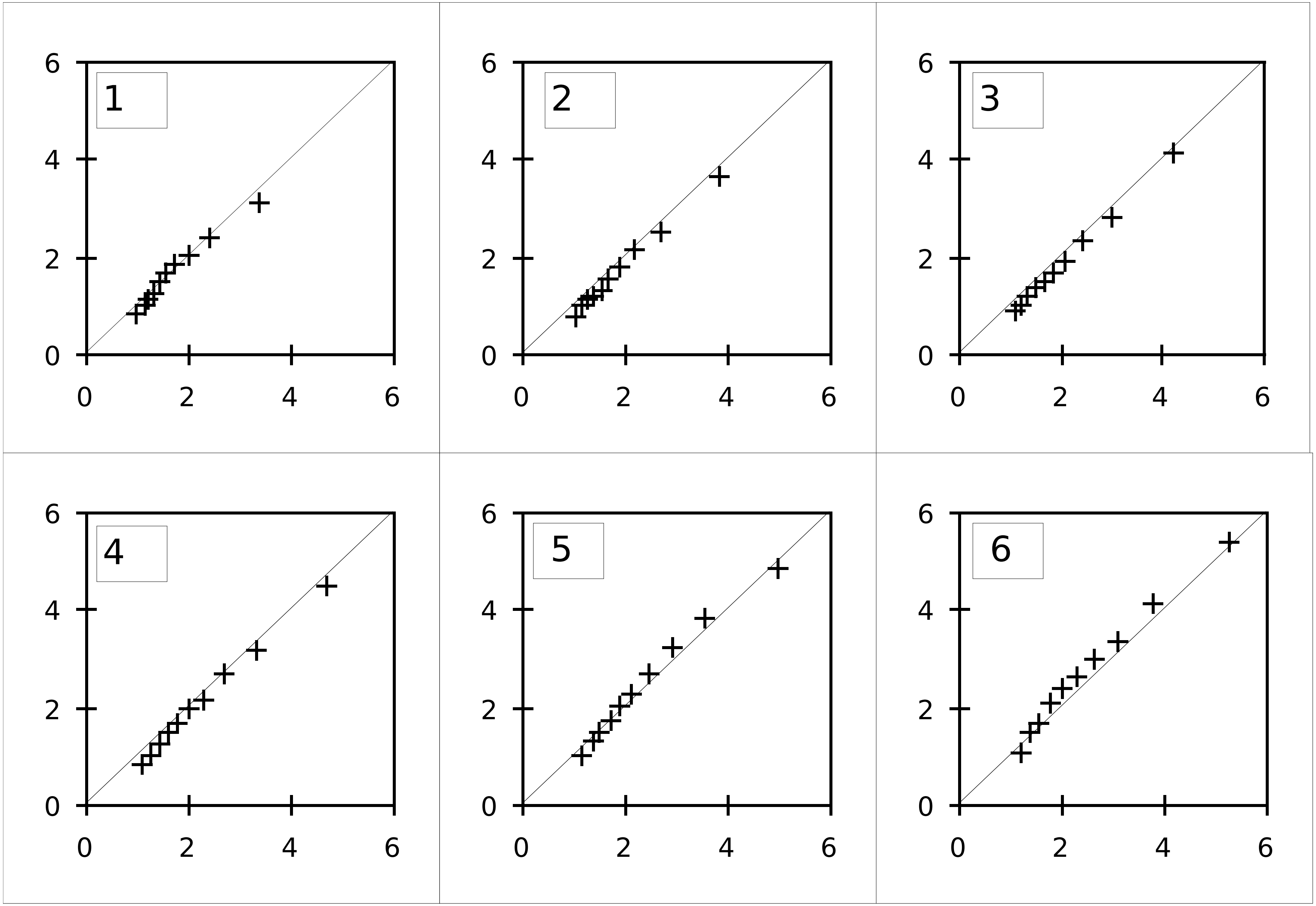


a)
Three-choice
Novel Elements
P[a:a,b,c]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8
7
6
5
4
3
2
1
0
Category-appropriate Elements
c)
q
q'
q statistic
2.5
2
1.5
1
0.5
0
-0.5
8
7
6
5
4
3
2
1
0
Category-appropriate Elements

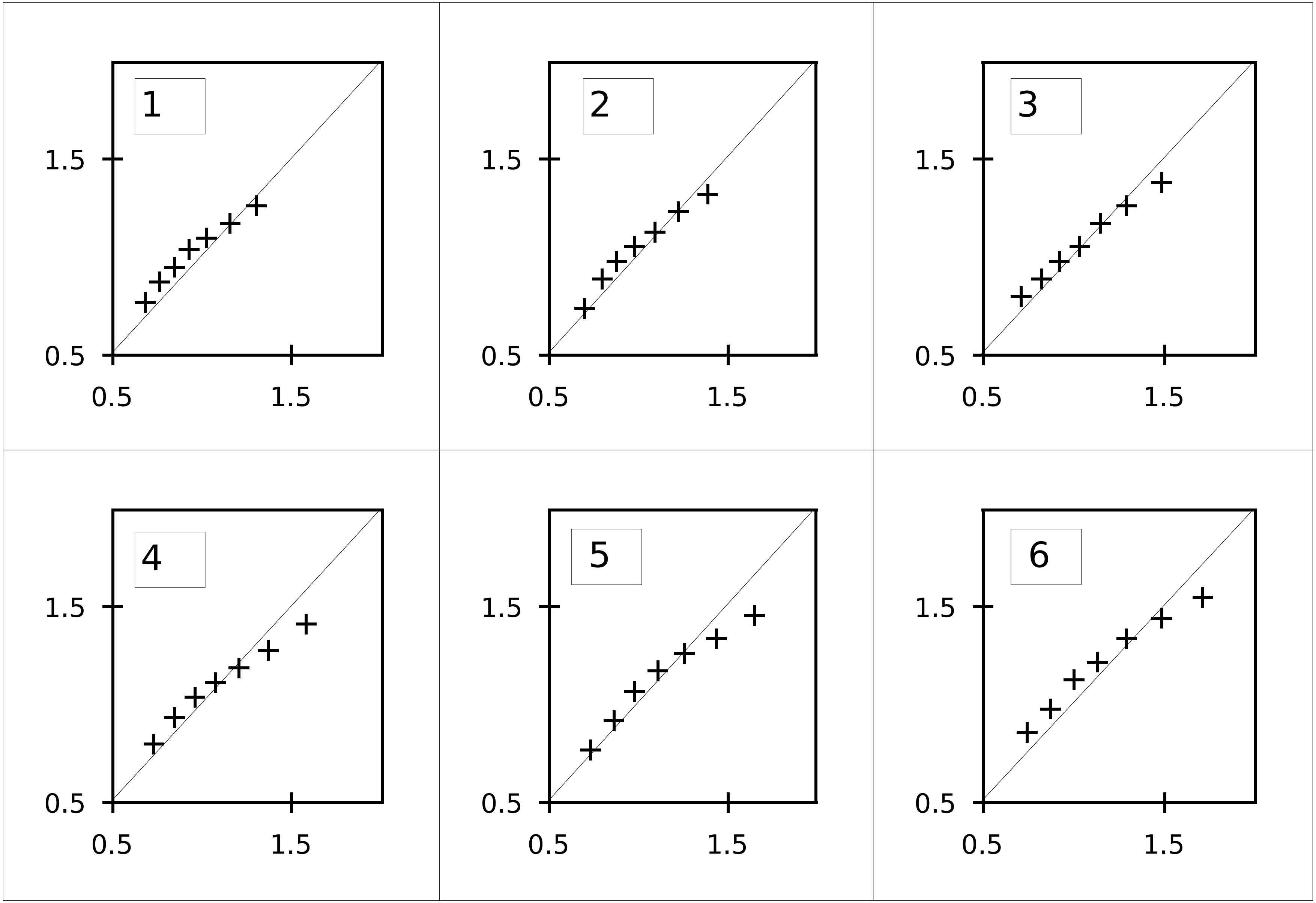
Time Pressure



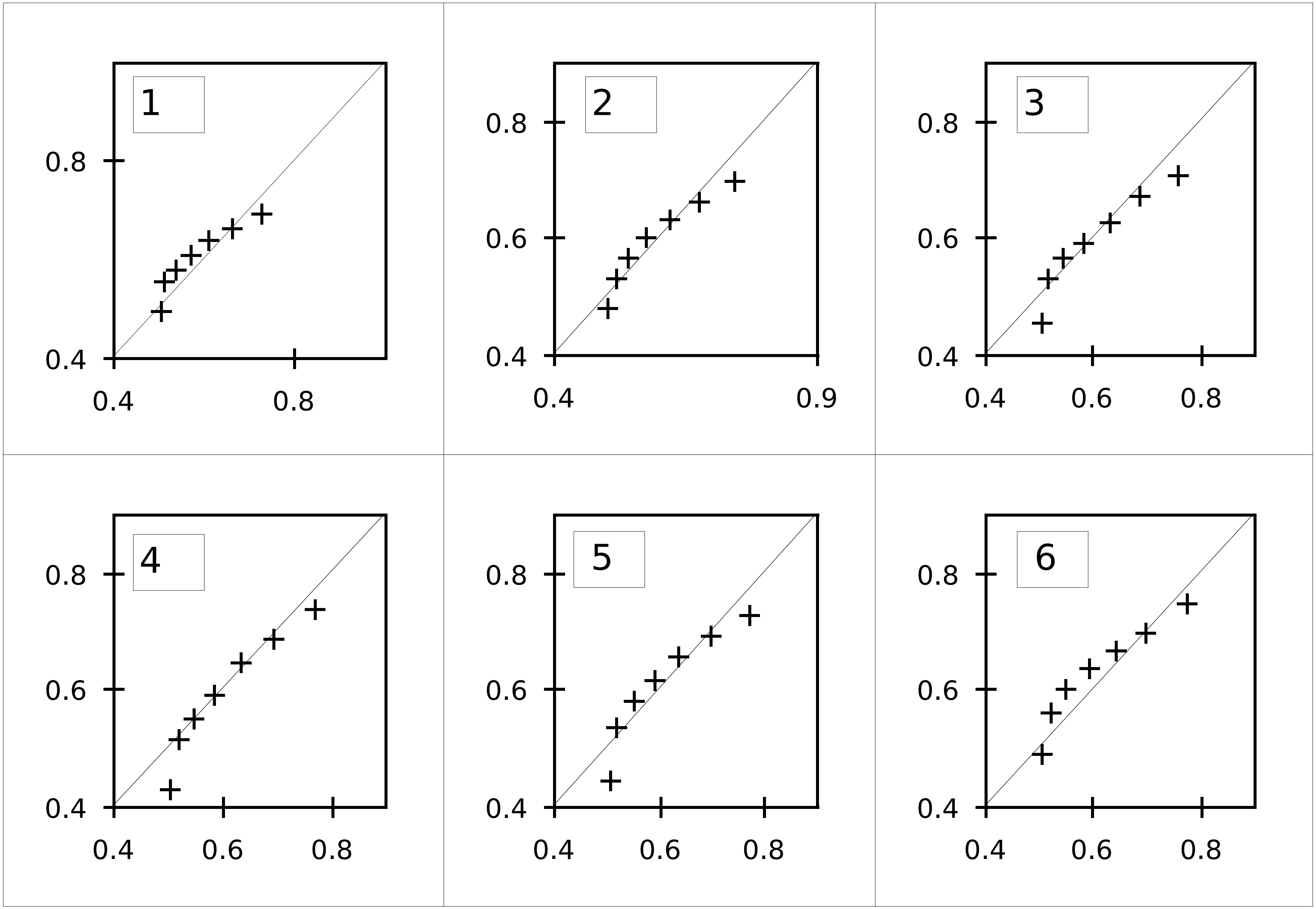
No Timeout



2.5s Timeout



1s Timeout


1
0.8
0.4
0.4
0.8
2
0.8
0.6
0.4
0.4
0.9
3
0.8
0.6
0.4
0.4
0.6
0.8
4
0.8
0.6
0.4
0.4
0.6
0.8
5
0.8
0.6
0.4
0.4
0.6
0.8
6
0.8
0.6
0.4
0.4
0.6
0.8

Summary

- The Ratio Rule is not a good theory of categorical decisions
- Could radically revise magnitude-based theories *a la* Tversky (1972)
- Or make the straightforward switch to Thurstonian choice